Formation Flight Near Libration Points: Survey and Recommendations

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Abstract

This paper gives a survey of some of the issues involved in formation flight of satellites near a halo orbit. An example of the use of such a formation is to act as a telescope for imaging interesting astronomical objects, such as extra-solar planets. Issues such as formation establishment, stability and dynamics of the formation, control and reconfiguration, coverage analysis, and inter-satellite communications are discussed. Because of the multidisciplinary nature of the tasks involved, an approach that involves several areas of research is needed.

I. INTRODUCTION. Formation flight near libration points offers a unique opportunity to establish a long-baseline imaging capability. Such capabilities are obviously of great long-range importance for NASA.

The Challenge

Using the experience gained in the last few years with missions such as the Genesis Discovery Mission, we have learned a great deal about the dynamics near the Sun-Earth libration points L_1 and L_2 (see fig. 1). However, this knowledge has also revealed how subtle and sensitive the dynamics are. Formation flight is even more challenging, with its own unique problems of stability, formation reconfiguration, and many other associated issues, which have not yet been addressed. Some of these issues are also being considered in studies of formation flight in near-Earth orbits; cooperative studies will, therefore, be essential.

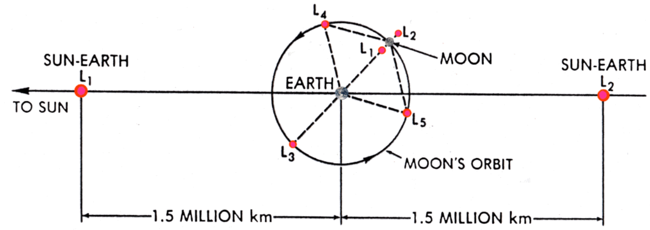

Figure 1. The Sun-Earth and Earth-Moon Lagrange points

Recent Advances

While the problem of formation flight near halo orbits is challenging, great strides have been made in the last decade concerning the dynamics and control of these orbits. The orbital structure of libration point orbits is now much better understood. Simple example constellations in loose formation have been constructed. Nonlinear control and optimal control methodology have been introduced into the problem with positive results. Deeper understanding of the entire phase space region near the libration points have been achieved.

Feasible, but More Work Needed

The problem now appears tractable, but much work remains. A multi-disciplinary approach combining traditional orbital dynamics techniques, dynamical systems theory, control and optimal control methods, with advanced numerical software is needed to solve this challenging problem. This paper provides a road map with our recommendations for the development of this technology.

II. Major Challenges. We first list what we consider to be the major challenges to formation flight near a halo orbit. After listing the problem areas, we discuss the analytical and computational techniques that we feel represent the proper approach to addressing these difficulties. We believe that addressing these problems requires an integrated approach to mission design using concepts and tools that are relatively new to the field of space flight.

The challenges to formation flight near a halo orbit are divided into three categories: trajectory design, formation management, and performance evaluation.

Trajectory Design

There are four major problem areas in trajectory design.

<u>Nonlinear trajectory design</u>. The complex dynamics of the three-body problem raises special challenges to trajectory design in the libration point regime. Other missions such as near-Earth missions, or even the Voyager and Galileo multiple flybys, are well approximated by two-body problems, which are fully integrable. This enables analysis using the "patched conics" or "multi-conics" techniques developed at JPL. Conic segments provide excellent approximations to the final trajectories along with estimates of propulsion and power requirements. For missions in which this approximation is not valid, trajectory design requires greater sensitivity to the underlying dynamics as well as appropriate computational tools.

<u>Low-thrust trajectory design</u>. The performance requirements of many formation flight missions, with regard to both formation maintenance and propellant consumption, necessitates the use of low-thrust trajecto-

ries and control paradigms. The design of low-thrust trajectories in nonlinear regimes remains an open problem. Primer vector theory, even for the impulsive case, is not immediately extensible from the two-body problem to the three-body problem. Existing numerical algorithms suffer from the numerical sensitivity of the underlying problem. Brute force approaches have also not proved successful.

Formation trajectory design. To create a satellite formation, one must design and generate the individual trajectories that each satellite must follow. The natural dynamics of the orbital mechanics must be exploited to provide an efficient trajectory. In many instances, continuous thrusting must be provided to achieve precision formation. In other instances, one may need only the relative locations of the satellites to high precision (using, for example, laser interferometry) and software could make up for the changing shape of the formation as long as it is relatively stable, without secular drifts. This problem combines the difficulties of the previous two problems and is extremely challenging. To date, other than the simplest case of two satellites following one another in the same circular orbit, there has been little work in precision formation flying, especially in nonlinear regimes.

Launch deployment. Once a formation has been designed, the launch deployment is an important next consideration. For formations with more than two or three satellites, multiple launch vehicles may be needed for deployment. The launches could be separated by significant periods of days or weeks, which may cause the spacecraft to be separated by great distances. Additional propulsion for the follow-on spacecraft to rendezvous with previously launched spacecraft would require significant amounts of propellant. In the case of quasi-halo orbits, the orbit phasing poses similar problems (fig. 2). The launch deployment is extremely important since in general, an inefficient launch strategy will typically be extremely costly in terms of the propellant requirements and the mission operations cost to repair the problem.

Formation Management

Formation design consists of two parts: the design of the individual trajectories constituting the formation and the design of the controller, which maintains the formation. Although the first part has already been described, the trajectory design and the formation control cannot really be separated. For the sake of this paper, we separate the control problem and present it in this subsection. We distinguish two types of formation control, loose and precision control. By "loose formation," we mean a satellite formation whose shape is not required to be "exact." For example, the simultaneous in-situ measurement of the magnetosphere would require a loose constellation scattered all over the magnetosphere. Here, a precise configuration is not required. However, proper scattering of formation is still complicated by the underlying nonlinear dynamics, as is performance estimation.

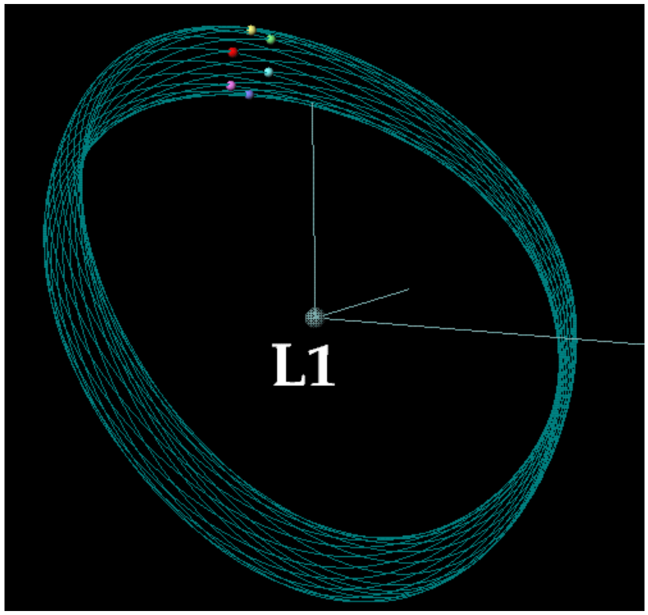

Figure 2. Quasi-halo orbits (Barden and Howell 1998)

For interferometry applications, the shape and orientation of the formation must be known and maintained precisely (or be managed in software) for sufficient time to collect the photons from distant astronomical objects. The control of this type of precision formation is an extremely challenging problem. Moreover, the control required to maintain precision formation in the libration regime is also difficult to estimate. As discussed earlier, estimation of the necessary control effort cannot be achieved by traditional conic-type approximations. Thus, designers are currently lacking a control paradigm for formation maintenance as well as the performance metrics used to evaluate design trades between different formations. This performance estimate is important, otherwise inferior orbit options could be selected instead, which would compromise the overall mission performance.

A serious issue not addressed by this paper is the autonomous on-board navigation and control for formation maintenance. Once a formation maintenance control algorithm has been produced, the engineering required to realize this control on-board the spacecraft in an autonomous manner needs to be carefully analyzed and verified. The design of the control must take into consideration what is achievable on-board the spacecraft.

The autonomous control issue highlights the multidisciplinary nature of this problem. The trajectory design, navigation approach, spacecraft dynamics, and control for the formation flight problem are not separable. In order to understand the problem and produce a workable solution, an integrated approach combining all of these fields is necessary.

Performance Analysis and Metrics

As with any engineering system, meaningful measurements of system performance are necessary for design evaluation. Defining and computing these metrics is frequently nontrivial, and often requires additional tools. We consider four key metrics of importance to any observatory or mapping missions like TPF or SIM.

<u>Coverage analysis</u>. We use the term "Coverage Analysis" in the widest sense possible in this context. We include here the various mission geometry constraints imposed by the requirements and design of the instruments, the spacecraft, and mission operations system. Loosely speaking, the product of the coverage analysis is an estimate of the amount of data collected. Based on this metric, one could then make quantitative trade studies and compare different designs.

<u>Propulsion requirements</u>. The propulsion requirement is a critical performance metric for formations as it drives the mass of the spacecraft. This is particularly difficult to analyze where low thrust trajectories are concerned as indicated earlier. Many of the estimation techniques for impulsive maneuvers are not applicable to low thrust trajectories, even in benign dynamical regimes. In the case of highly nonlinear dynamical regimes, this adds to the complexity of the problem. Estimating propulsion requirements is a challenge both for design of the overall trajectory as well as maintenance of the formation.

<u>Power requirements</u>. The power requirement is also a critical performance metric for formations, particularly if low-thrust propulsion is used, since the battery size also contributes significantly to the mass of the spacecraft.

<u>Spacecraft Mass</u>. The above requirements all drive the spacecraft mass. This is the ultimate performance metric which depends not only on the trajectory design, control algorithm, propulsion and power subsystems, but also on the launch vehicle capability and launch deployment strategy.

Common to all the problems discussed above is the lack of techniques and tools for analyzing highly nonlinear and non-integrable problems. Tools used to design trajectories such as those of Voyager and Galileo simply cannot be extended to missions that cannot be approximated by a two-body problem, as discussed earlier. Furthermore, the complexity and novelty of formation flight demand the development of new analytical and computational tools around which new missions, such as the TPF mission, can be designed.

III. SURVEY OF WORK RELATED TO FORMATION FLIGHT NEAR A HALO ORBIT. As we have discussed, formation flight around a halo orbit combines the need for formation maintenance with the complex dynamics of the three-body problem. We believe that addressing this challenge requires an integrated approach using concepts and tools that are relatively new to mission design, such as dynamical systems theory and geometric mechanics combined with optimal control. We will provide a survey of advanced work in these fields, which are crucial for a successful design of formation flight near a halo orbit.

Dynamical Systems Theory

Dynamical systems theory is the geometric theory of differential equations developed by Poincaré at the turn of the 20th century to study the three-body problem, which is precisely the setting for the TPF mission. Some of the most important work in the application of dynamical systems theory to space missions has been done in Barcelona, Purdue, JPL, and Caltech.

Invariant manifolds. One of the most useful aspects of dynamical systems theory to space missions design is the role of invariant manifold theory. These manifolds are surfaces in the trajectory design space consisting of global families of trajectories that wind on and off periodic orbits such as halo orbits. The power of dynamical systems theory is that it enables us to compute and visualize these surfaces so that we can obtain a map of the orbit design space and hone in on the specific trajectories useful to a particular mission. An example is the design of the trajectory for the Genesis Discovery Mission (Howell, Barden, and Lo 1997; Lo et al. 1998). Genesis is the first mission designed using dynamical system theory. The entire trajectory after launch requires no deterministic maneuver to capture into halo orbit for two years and automatically return to Earth. Without this approach, this mission could not be accomplished within its propellant budget.

Dynamical channels and efficient deployment. Based on a deeper understanding of the interactions between these invariant manifolds, a network of "interplanetary superhighways" (called dynamical channels) governing the material transport in the solar system have been found in Koon et al. 2000a. A number of new techniques for constructing spacecraft trajectories with desired characteristics have also been developed using this methodology. These techniques have been used to design a "Petit Grand Tour" of Jovian moons and an Earth-to-Moon lunar ballistic capture mission which uses less fuel than Hohmann transfers (Koon et al. 2000b). This network of dynamical channels can be a key component in designing low-fuel paths for deployment of a spacecraft constellation to and from Earth, including low-thrust trajectories.

Quasi-halo orbits and formation flight. For the specific problem of formation flight near a halo orbit, there exist two distinct approaches to the computation of these orbits, called quasi-halo orbits, which form the basis of any constellations in this region of space. Gómez, Masdemont, and Simó (1997) discovered these quasi-halos around a halo orbit on which the motion is quasi-periodic with two basic frequencies. The longitudinal frequency corresponds to the frequency of the baseline halo orbit, and the latitudinal frequency (which depends on the size of the torus on which the orbit winds) corresponds to the motion winding around the halo orbit. Their method, based on series expansion, provides a good control over the characteristics of the quasi-halo orbits that it computes and can be used to investigate the natural dynamics of these orbits. Since it is virtually impossible to design a formation in flight near the libration points without a good understanding of this basic dynamics, these tools will be important both in studying the feasibility and in designing the actual formation trajectory for the constellation near a halo orbit.

Barden and Howell (1998, 1999) developed a different method to compute these same orbits. After obtaining a linear approximation of a quasi-halo, they used it as an initial guess for a differential corrector which patched a number of integrated trajectory segments into a continuous quasi-periodic solution. Their method provided some insight into how these quasi-halo orbits are related to the halo orbit and allowed them to design a ring formation near a halo orbit. Since it is impossible to design a formation in flight near the libration point without a good understanding of the nearby orbital dynamics, the recent theoretical and numerical advances mentioned above have raised the possibility of flying multiple spacecraft in a controlled formation near a halo orbit.

Geometric Mechanics and Control Theory

Geometric mechanics is the study of the behavior of mechanical systems, i.e., systems that admit a Lagrangian or Hamiltonian formulation. One of the central notions of geometric mechanics is the representation of those systems in a framework which is coordinate-independent and which exposes the underlying structure of the equations. This framework has proven to be extremely effective in a wide variety of problems for dynamical analysis.

Recent advances in control theory have led to renewed interest in geometric mechanics and increased its relevance to the engineering community. In the sections below, we describe some of the tools of geometric mechanics that are potentially useful in formation mission design and analysis.

Shape dynamics, geometric phase and formation flight. One of the main achievements in mechanics has been the development of the concept of shape dynamics and the geometric phase. Using tools from geometric mechanics, a dynamical system can be systematically partitioned into shape (i.e., internal) and external dynamics, which are driven by the shape dynamics. Note that for many problems, isolating the shape space is far from trivial. This partitioning leads to the concept of geometric phase, meaning the motion of the overall system due to cyclic motions within the shape space. Understanding the behavior of the geometric phase can lead to control paradigms whereby a trajectory of the overall system may be modified through the appropriate choice of shape deformation.

The relevance of this shape to formation flight can be understood by realizing that a spacecraft formation acts like a deformable body consisting of a system of rigid bodies that are connected via control and information exchange. This system possesses a shape space, and the formation shape is deformed by the natural motion of the spacecraft as well as the control effort exerted to maintain the formation. Using the notion of geometric phase, one can understand how internal formation maintenance affects the overall trajectory of the formation. If, for example, one wished to reconfigure the formation without veering off the desired trajectory, understanding the behavior of the geometric phase would be essential.

Controllability and trajectory design. The geometric formulation of mechanical systems is useful for determining the controllability properties of mechanical systems, i.e., the ability to move a mechanical system from one point to another through appropriate choice of controls. In particular, the ability to steer a mechanical system from one equilibrium point to another has been studied by Lewis and Murray (1999). Sufficient conditions are given for "equilibrium controllability" of Lagrangian systems that indicate how the interaction between the control forces and the metric properties of the system combine to give controllability. These results allow controllability of a single satellite as well as controllability of the entire cluster (shape, position, and orientation) to be evaluated. The importance of the controllability structure for motion generation is twofold. First, controllability can be used as a necessary criterion in designing a mechanical system. For example, if we are interested in being able to control the position, orientation, and shape of a satellite formation by actuating every possible degree of freedom, this controllability structure can be studied to insure that this is possible. Second, the controllability conditions can serve as a guide for constructing motion sequences that move the system from one configuration to another.

Optimal Control

The performance requirements of many-formation flight missions, with regard to both formation maintenance and propellant consumption, necessitate the use of low-thrust trajectories and control paradigms. The design of low-thrust trajectories in nonlinear regimes remains an open problem.

Theoretically, one of the most favored approaches is to use optimal control in generating the low-thrust trajectories. But numerically, there exist a number of difficulties. Primer vector theory, even for the impulsive case, is not immediately extensible from the two-body problem to the three-body problem. Existing numerical algorithms would not converge due to the sensitivity of the three-body dynamics. In the ongoing effort in using optimal control to study certain JPL orbit transfer problems, Koon et al. 2000a have proposed to tackle these difficulties from two fronts: (1) to explore the use of "direct method" for solving optimal control problem, and (2) to merge optimal control and dynamical system theory.

The direct method in optimal control algorithms. Essentially two approaches have emerged over the past four decades to solve the optimal trajectory problem numerically: the indirect and direct methods. In the indirect method, the necessary conditions for optimality are derived using the techniques of calculus-of-variations or the Pontryagin Maximum Principle. The main drawbacks of the indirect method are the numerical sensitivity of the Euler-Lagrange equations and the frequent occurrence of discontinuities in the optimal control. In the direct method, the optimal control problem is approximated by a discrete optimization problem. This avoids the numerical difficulties of solving the Euler-Lagrange equations. With the development of sophisticated sequential quadratic programming software to solve the resulting optimization problem, there has been a new explosion of research on the direct method in the last few years. The resulting numerical algorithm is very robust.

Merging optimal control and dynamical systems theory. As usual, for any numerical algorithm, a good initial guess is vital, especially if the problem is very sensitive numerically. Dynamical systems theory can provide geometrical insight into the structure of the problem and even good approximate solutions. For example, in finding low-thrust optimal transfers to L_1 halo orbits in the Sun-Earth system, it is important to know that the invariant manifolds of the halo orbits extend to the vicinity of the Earth and any trajectory on these manifolds can be used as a super-highway for free rides to and from the halo orbits. Clearly, this theoretical insight and its derivative numerical tools can aid in the construction of superior initial guesses that lead to a convergent solution.

A deeper understanding of the dynamical structure of the three-body problem may suggest alternate formulations of the optimizing scheme that are based more on the geometry of the phase space. Instead of "numerically groping in the dark," algorithms could be developed with the natural dynamics built in, thereby yielding better convergence properties.

Optimal control and trajectory correction maneuvers. The two ideas mentioned above have been put to an initial test in the joint work between Caltech, JPL, and UCSB (the Computational Science and Engineering Group; Serban et al. 2000). This paper addresses the computation of the required trajectory correction maneuvers (TCM) for the Genesis mission to compensate for the launch velocity errors introduced by the inaccuracies of the launch vehicle.

Right after launch, before the spacecraft initial checkout activities have been completed and the spacecraft performance and orbit have been characterized by the flight team, the performance of an early maneuver such as TCM1 is both difficult and risky. It is desirable to delay TCM1 as long as possible, even at the expense of expenditure of the ΔV budget. In fact, Genesis would prefer TCM1 be performed at 2 to 7 days after launch, or even later. However, beyond Launch+24 hours, the correction ΔV based on traditional linear analysis can become prohibitively high.

The desire to increase the time between launch and TCM1 drives one to use a nonlinear approach, based on combining dynamical systems theory with optimal control techniques. Two similar but slightly different approaches one can use to get an optimal maneuver strategy that fits within the ΔV budget of 150 m/s allotted to TCM. (1) HOI technique: use optimal control techniques to re-target the halo orbit with the original nominal trajectory as the initial guess. (2) MOI technique: target the stable manifold. Both methods yield good results using the software COOPT which is based on the direct method and developed at UCSB.

We feel that COOPT or similar software and the methods of optimal control and dynamical systems can be used for many missions in the future. It will be an essential tool for designing formation flights near a halo orbit.

Software Tools

While the development of theoretical tools and algorithms are crucial, ultimately, it is the software that enables the actual implementation of a space mission using formation flight. Currently, there are a number of tools that address the design of libration point missions. They include commercial packages such as STK/Astrogator, Free Flyer, and NASA-developed tools such as Swingby. Purdue University and Barcelona University also have software packages addressing the trajectory design in libration point orbits. Leveraging on the university work, JPL is collaborating with Caltech, Purdue, and Barcelona to develop LTool. Although, these tools address some aspects of the software infrastructures necessary for work on formation flight, none of them are able to address the full problem. Of course, this is due in part to the immaturity of the theoretical and algorithm work in this field. As an example of these tools, we describe LTool.

<u>LTool</u>. LTool is JPL's Libration Point Mission Design Tool currently under development in support of the mission design and operations of liberation point missions. The Genesis Discovery Mission is LTool's first customer. The driving requirement of LTool is to enable the users to quickly develop modules or reconfigure existing components to solve new problems in space mission design using cutting edge semi-analytical methods including dynamical systems theory and optimal control theory. The primary goal is to provide the user with as much control and flexibility as possible but in an organized environment with persistent astrodynamical objects, multi-threaded computation capabilities, and

advanced visualization capabilities. LTool provides an interactive command line interface with scripting capabilities; user definable graphical user interfaces are planned. In addition to interactive stereo 3D visualization capability, LTool will also provides 3D visualization in a semi-immersive virtual reality environment in collaboration with the Caltech Graphics Group using the Graphics Group's Responsive Workbench.

COOPT. To design and control an energy-efficient formation, the use of optimal control tools is essential. An example of a state-of-the art software package for optimal control and optimization of differential equations is COOPT, developed at UCSB. COOPT is based on the direct method in optimal control theory. It is excellent in providing an optimal solution efficiently. Moreover, it also provides parametric studies of the cost function as a function of the control parameters. For instance, in the TCM1 study cited above (Serban et al. 2000), COOPT not only provided the optimal time and location to perform the TCM1, it also provided the sensitivity of the ΔV cost to variations in the timing of TCM1. However, other optimal control software packages that exploit the mechanical structure of a system are also under development and it is possible that even more efficient techniques are possible.

IV. Recommendation. The problems outlined in the above sections represent some of the most difficult problems in modern astrodynamics and celestial mechanics. Their solutions will require hard work perhaps for the next decade. As we have emphasized, the increasing complexity of mission requirements, such as formation flight near a halo orbit, has rendered the old trajectory and navigation tools inadequate. Successful formation design requires an integrated approach using concepts and tools listed above, which are relatively new to the mission design community. Although these problems will require long-term study for complete solutions, in the near-term, it is now possible and highly desirable to provide first-order performance estimates to establish bounds on the requirements for propulsion and power. Using these estimation tools, TPF planners can quickly determine the feasibility of various formations in different regimes of orbit design space, including the L2 regime. However, to carry out the detailed analysis, design, and engineering necessary to implement an actual mission in formation flight using libration point orbits, a lot of hard work remains. In the remaining sections, we present a road map for the development of this enabling technology.

Our recommendation is structured around two main goals:

1. Develop metrics and evaluation tools for formation flight around a halo orbit and apply them to the TPF mission in the near-term
2. Develop astrodynamic technology and tools necessary to design and fly the TPF mission in the long-term

Develop Metrics and Evaluation Tools

This task should include:

1. Provide initial kinematic orbit design and selection
2. Define metrics and evaluation procedures
3. Develop computational and visualization software tools for the above items

For instance, one can select a "mother" orbit, which the "daughters" follow. In the initial study of the L2 TPF formation, a halo orbit's success as the "mother" orbit. The daughter orbits following in formation will not be natural orbits of the gravitational dynamics, but are kinematic trajectories whose positions and velocities are completely defined by the specified flight formation and the mother orbit. The accelerations of the entire formation can be computed and the energy required to power this formation may be computed accordingly. Other "mother orbits" that may be segments of quasi-halo orbits can also be studied in the same way. Although this "brute force method" may be unsuitable for design purposes, it provides a first order estimate of the upper bounds for the energy required for formation flight. Furthermore, such a solution may be used as an initial guess to trajectory optimization programs for possible energy reduction. The development of software that allows one to replace precision formation flying with precision knowledge of the relative locations of the members of the formation also greatly affects the mission requirements.

It is expected that the preceding ideas may be too crude and too far from the optimal to be of general use. Nevertheless, it will be extremely useful as a metric for system performance. Also with the use of methods like genetic algorithms, such an initial guess may even produce a near-optimal solution. In engineering, we are not seeking the global minimum, though that would be nice, we seek a solution that fits within our cost function limits.

Develop Astrodynamics Tools

This task consists of the development of theoretical foundations, computational algorithms, software analysis, and advanced visualization tools. What follows is a more detailed breakdown for various fields of study.

1. Development and applications of quasi-halo theory.
   - New formation development
   - Launch and transfer to formation
   - Rendezvous problem in libration point region
2. Geometric mechanics framework for control.
   - Apply shape theory to point-mass N-body case
   - Apply shape theory to rigid body N-body case

3. Applications of optimal control.
   - Apply optimal control direct methods to low thrust
   - Merge optimal control with libration point dynamics
   - Explore formation management with low thrust control

Dynamical systems theory. In order to design the TPF formation, a greater understanding of quasi-halo orbits is essential. TPF design requires the study of several formations beside the current ring formation. A second equally important problem is that of launching and deploying the constellation into the proper formation. Both problems need to be examined.

Geometric mechanics and control. For the application of geometric mechanics, we recommend to first study the formation as an ensemble of "point-mass" spacecraft without regards to the attitude of the individual spacecraft. This greatly simplifies the problem by separating the individual spacecraft body dynamics from that of the formation. After having a better handle on this simpler problem, one can proceed to the full problem where the ensemble of spacecraft is now viewed as a collection of finite bodies and not just as points.

Optimal control. The techniques embodied in the TCM1 study need to be developed and applied to the design of low thrust trajectories in the regime of libration point dynamics. After having a better understanding of the dynamics of quasi-halo orbits and formations, one can then explore the formation management near a halo orbit.

Advanced visualization. We mention the need for advanced, responsive visualization tools in the design and analysis process. A simple example illustrates the point: by plotting and visualizing a function, f(x,y) as a 2D surface, one can easily find its maxima and minima without computing the gradient or the hessian. One does not even need to know their mathematical meaning! In the case of higher dimensional objects, the use of interactive stereo 3D visualization, animation, virtual environments, and graphics responsive to manual manipulation and sense of touch will add immeasurably to the design and analysis process. Preliminary work with the Caltech Graphics Group's Responsive Workbench have surprised even seasoned trajectory designers at the new insight and sense of intuition provided by some of the prototype tools.

Integrated approach. Once the techniques in dynamical systems theory, geometric mechanics, optimal control, software and graphics tools have been individually advanced within an overall framework, one can then combine them to produce new techniques for mission design. While the integration of these applications will be the most difficult and challenging task, this systematic approach which draws on the combined strength of various fields will produce the desired solution. In this instance, the integrated system will be much greater than the sum of the parts.

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